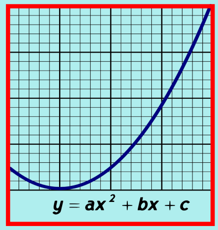
Math 25

Fall 2017

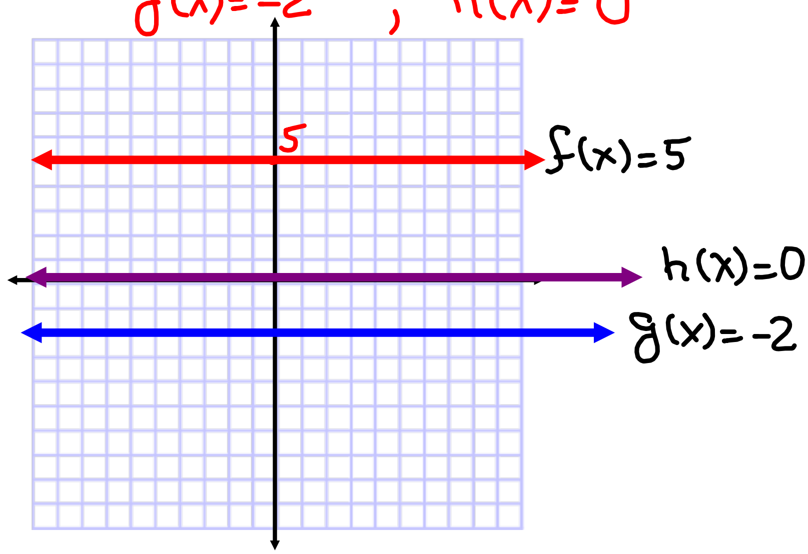
Lecture 2



Cont. with functions

$f(x) = b$ constant function

$f(x)=5$, $g(x)=-2$, $h(x)=0$



$f(x) = mx + b$ Linear Function

Slope m

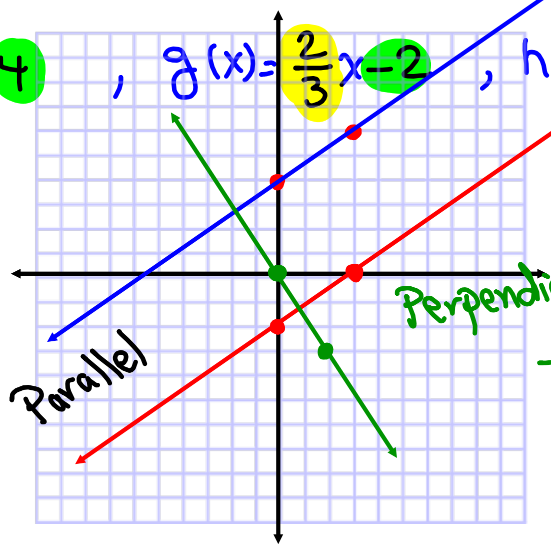
Y-Int $(0, b)$

$y = mx + b$

$f(x) = \frac{2}{3}x + 4$, $g(x) = \frac{2}{3}x - 2$, $h(x) = \frac{-3}{2}x$

Increasing

Y-Int $(0,0)$



$2/3 = 2/3$ Parallel

Perpendicular

$\frac{-3}{2} \cdot \frac{2}{3} = -1$

How to find eqn of a line

1) No slope, undefined slope → vertical line
$x = a$

2) Zero slope, $m = 0$ → Horizontal line
$y = b$

3) otherwise → use point-slope formula

$y - y_1 = m(x - x_1)$

Recall $m = \frac{y_2 - y_1}{x_2 - x_1}$

we always prefer final ans in
$y = mx + b$

Find equation of a line that contains $(-3,2)$ and $(0,4)$.

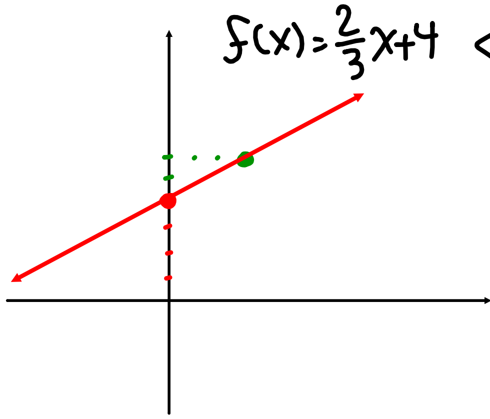
1) Find slope $\quad m = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{2-4}{-3-0} = \dfrac{2}{3}$

2) Use point-slope $\quad y - y_1 = m(x - x_1)$

$$y - 4 = \frac{2}{3}(x - 0)$$

$$y - 4 = \frac{2}{3}x$$

$$y = \frac{2}{3}x + 4$$



Find eqn of a line that contains $(-3,4)$ and $(2,0)$. Ans. in Slope-Int form. Draw.

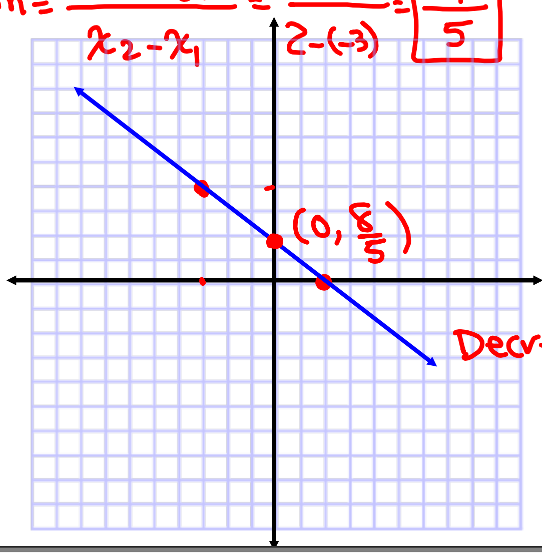
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{2-(-3)} = \boxed{\frac{-4}{5}}$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-4}{5}(x - 2)$$

$$y = \frac{-4}{5}x + \frac{8}{5}$$

$$\hookrightarrow f(x) = \frac{-4}{5}x + \frac{8}{5}$$



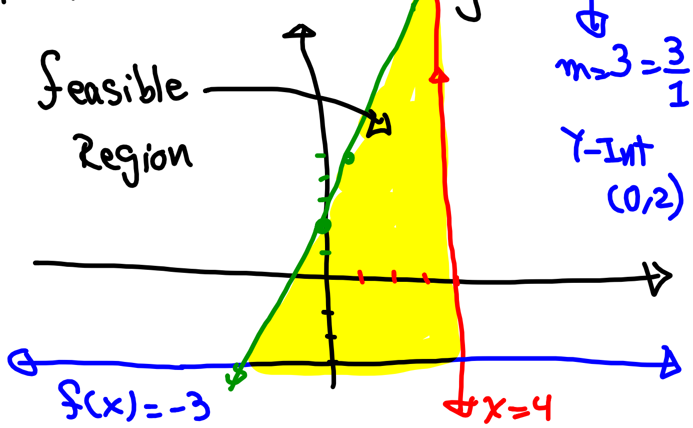
Decreasing

$m = \frac{-4}{5}$

Y-Int $(0, \frac{8}{5})$

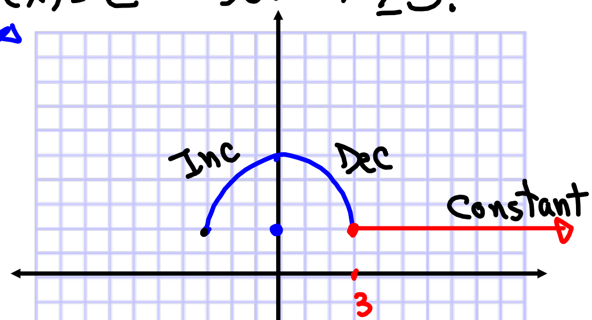
Graph $x=4$, $f(x)=-3$, and $g(x)=3x+2$.

Shade the region that is bounded by all three lines.



Graph the top half of the circle centered at (0,2) with radius 3 and $f(x)=2$ for $x \geq 3$.

Constant function



Increasing $(-3,0)$

Decreasing $(0,3)$

Constant $(3,\infty)$

Whenever discussing Inc., Dec., or constant use interval notation with (,).

Piece-wise defined function

$$f(x)=\begin{cases} x^2 & \text{for } x \le 1 \\ \frac{2}{3}x-4 & \text{for } x>1 \end{cases}$$

Recall $y=x^2$

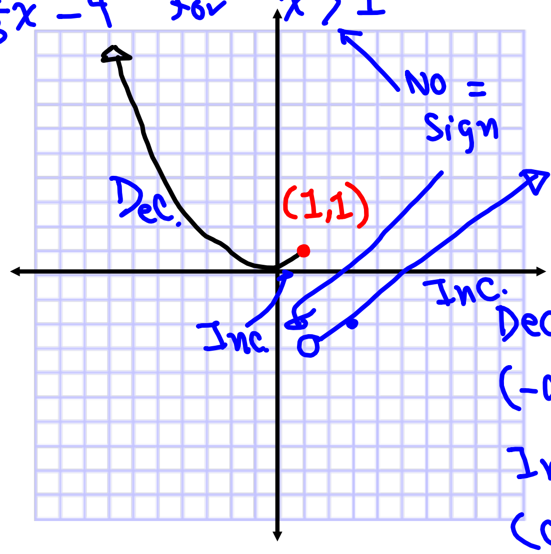
Parabola

$f(x)=x^2$

Parabola

x	y
0	
1	
-1	
2	
-2	

$f(x)=\frac{2}{3}x-4$



Decreasing
$(-\infty, 0)$

Increasing
$(0,1), (1,\infty)$

Graph

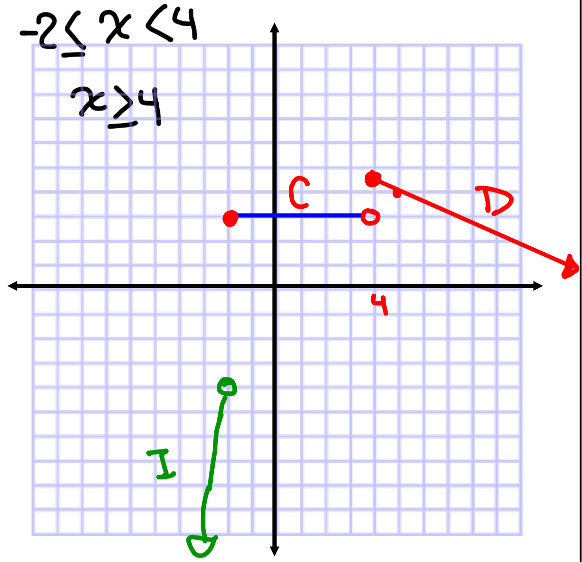
$$f(x)=\begin{cases} -x^2 & \text{for } x<-2 \\ 3 & \text{for } -2 \le x<4 \\ -\frac{2}{5}x+6 & \text{for } x \ge 4 \end{cases}$$

$f(x)=-x^2$

Increasing $(-\infty,-2)$

Constant $(-2,4)$

Decreasing $(4,\infty)$

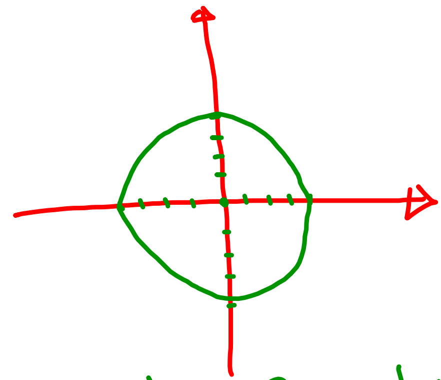


$x^2 + y^2 = 16$

Center $(0,0)$

Radius 4

Draw



Domain $[-4,4]$

Range $[-4,4]$

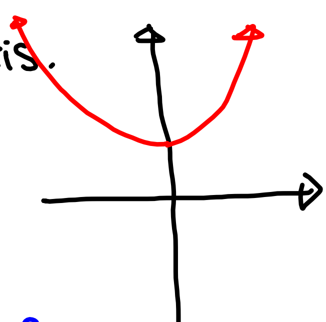
This graph is symmetric with respect to Y-axis, x-axis, and the origin.

Test for symmetry

1) Replace x with $-x$, simplify.

If we get same equation,

Graph will be symmetrix with respect to Y-axis.



$y = x^2 + 4$

$y = (-x)^2 + 4$

$y = x^2 + 4$

Same eqn.

Y-axis symmetry

2) repacle Y with $-y$, Simplify

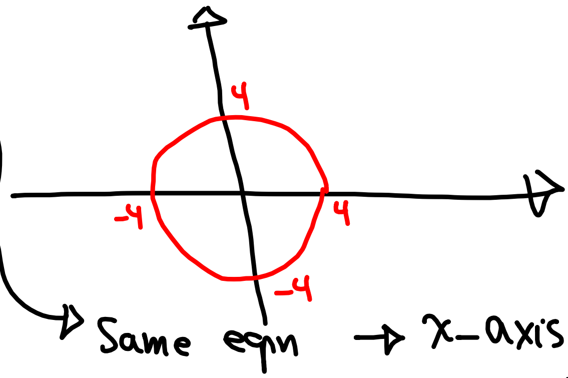
If we get Same eqn, we have

x-axis symmetry.

$x^2+y^2=16$

$x^2+(-y)^2=16$

$x^2+y^2=16$



Same eqn → x-axis symmetry.

3) If we replace x with $-x$, and y with $-y$, and Simplify

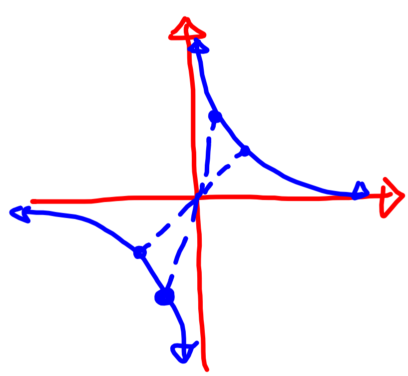
we have symmetry to the origin if we get Same eqn.

$xy=4$

$(-x)(-y)=4$

$xy=4$

Origin Symmetry



Check for Symmetry

$x^3 + |y| = 4$

$(-x)^3 + |-y| = 4$

$-x^3 + |y| = 4$

No Symmetry.

Y-axis symmetry

$(-x)^3 + |y| = 4$

$-x^3 + |y| = 4$

No symmetry

X-axis symmetry

$x^3 + |-y| = 4$

$x^3 + |y| = 4$

we have symmetry

Even, odd, or neither functions:

If $f(-x) = f(x) \rightarrow f(x)$ is an even function

$\rightarrow$ Symmetric with respect to Y-axis.

If $f(-x) = -f(x) \rightarrow f(x)$ is an odd functio[n]

$\rightarrow$ symmetric with respect to the origin.

otherwise it is neither.

$f(x) = x^4 + x^2 + 4$

$f(-x) = (-x)^4 + (-x)^2 + 4 \Rightarrow f(-x) = x^4 + x^2 + 4$

$f(-x) = f(x) \rightarrow$ even function

Y-axis symmetry

$f(x) = \frac{1}{x}$ $\quad f(-x) = \frac{1}{-x} = \frac{-1}{x} = -\frac{1}{x} = -f(x)$

$f(-x) = -f(x) \rightarrow$ odd function

$\rightarrow$ origin Sym.

$f(x) = x^3 - |x| + 1$

$f(-x) = (-x)^3 - |-x| + 1$

$= -x^3 - |x| + 1$

$f(-x) \neq f(x) \rightarrow$ Not even

$f(-x) \neq -f(x) \rightarrow$ Not odd

So $f(x)$ is neither.

No function is ever symmetric with respect to x-axis

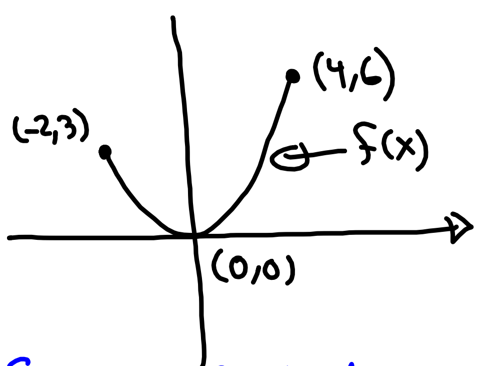
Transformations of functions $h, k > 0$

$f(x-h)$ moves $f(x)$ to the right h units.

$f(x+h)$ " " " " left h ".

$f(x)+k$ moves $f(x)$ up k units

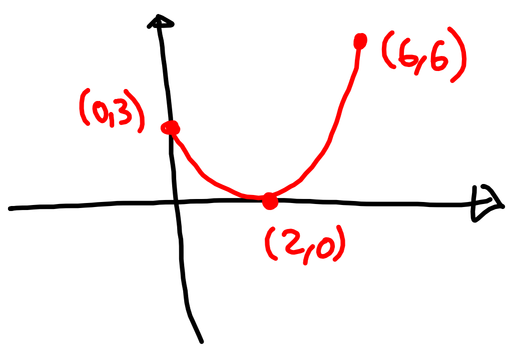
$f(x)-k$ " " down k units.



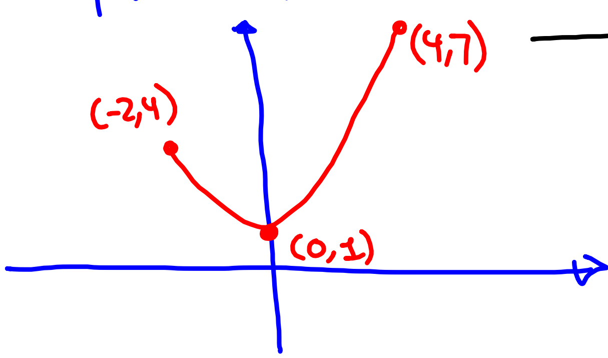
Graph

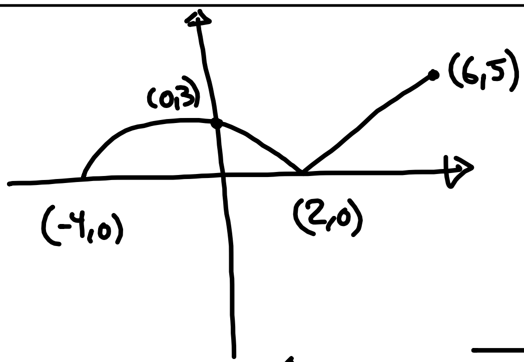
$f(x-2)$

Move right 2 units



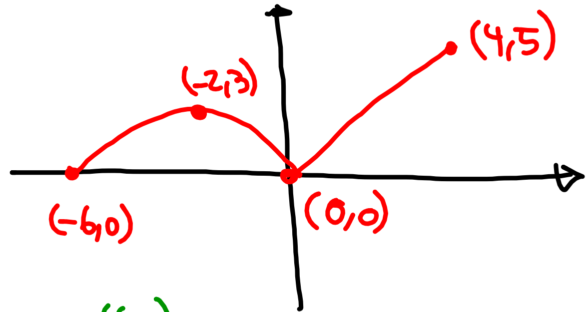
Graph $f(x)+1$

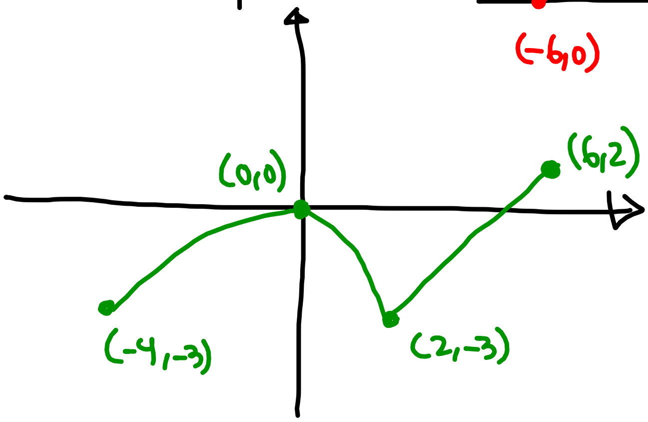




Graph

$f(x+2)$, $f(x)-3$



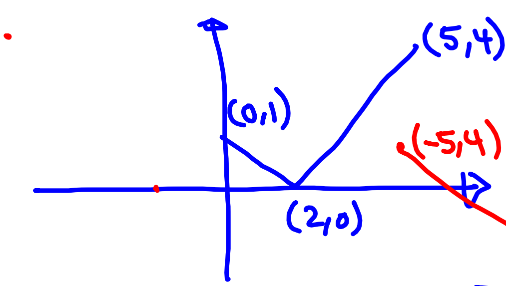


$f(-x) \to$ Reflection about Y-axis

$-f(x) \to$ Reflection about x-axis.

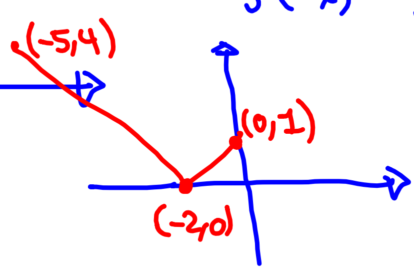
If $a>1$ $f(ax)$ compress horizontally

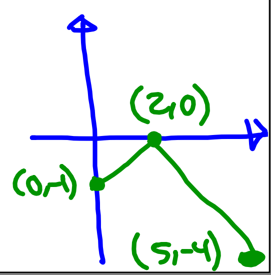
If $0<a<1$ $f(ax)$ stretch horizontally

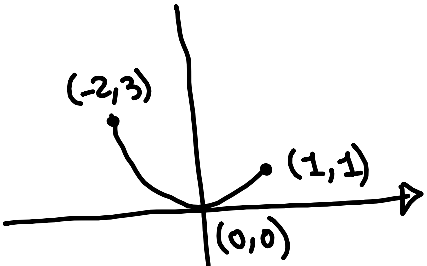


graph

$f(-x)$ & $-f(x)$







Graph

$f(2x)$

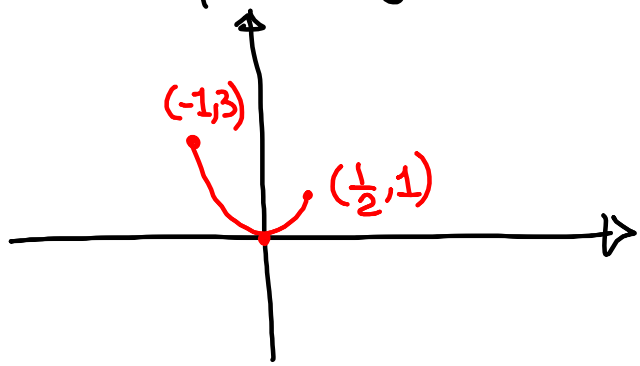
Compress by factor of 2.

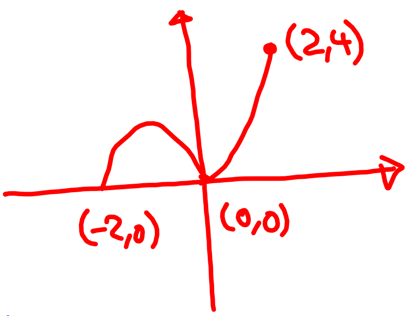
$2x=-2$

$x=-1$

$2x=0 \quad x=0$

$2x=1 \quad x=\frac{1}{2}$



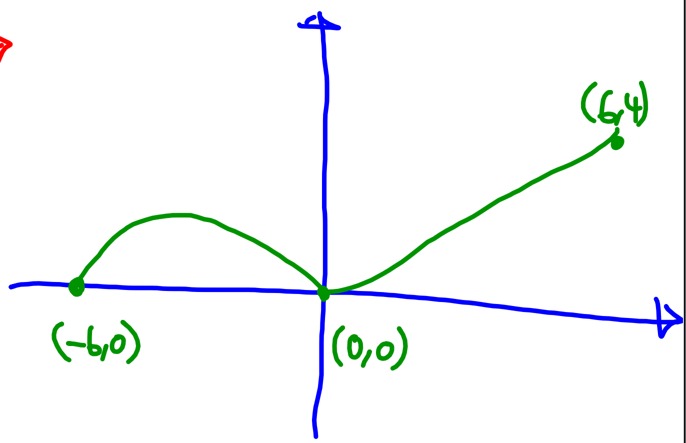


graph $f(\frac{1}{3}x)$

$\frac{1}{3}x=-2 \quad x=-6$

$\frac{1}{3}x=0 \quad x=0$

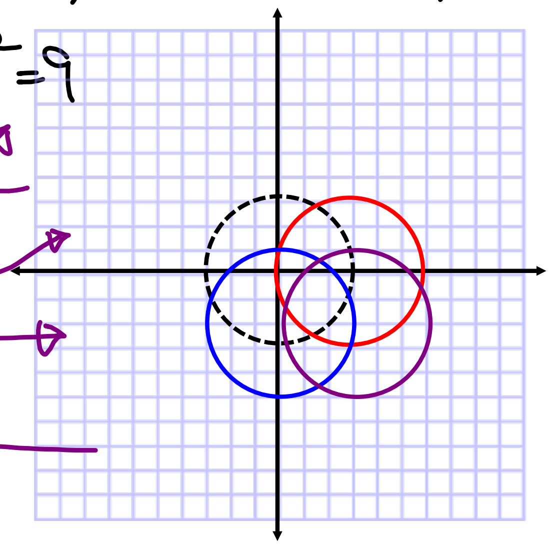
$\frac{1}{3}x=2 \quad x=6$



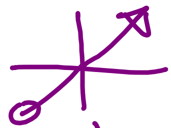
$x^2 + y^2 = 9$

Graph $(x-3)^2 + y^2 = 9$, $x^2 + (y+2)^2 = 9$,

and $(x-3)^2 + (y+2)^2 = 9$

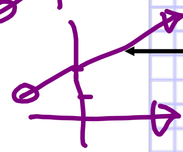


$y = x$



$y - 2 = x$

$y = x + 2$

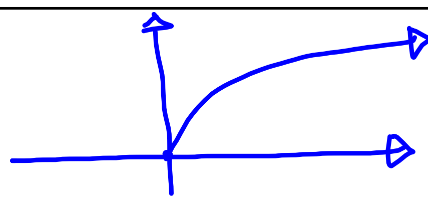


$y_1 = f(x)$

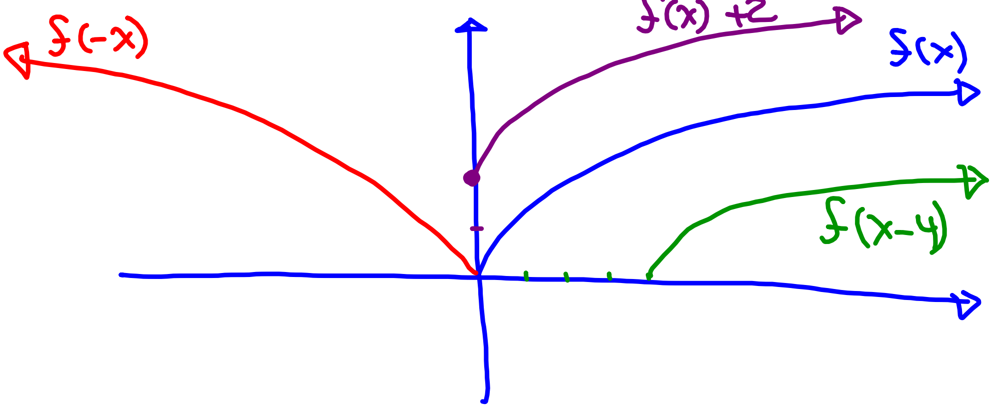
$y_2 + 3 = f(x)$

$y_2 = f(x) - 3$

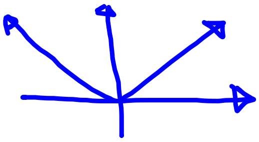
$f(x) = \sqrt{x}$



Graph $f(-x)$, $f(x-4)$, $f(x)+2$

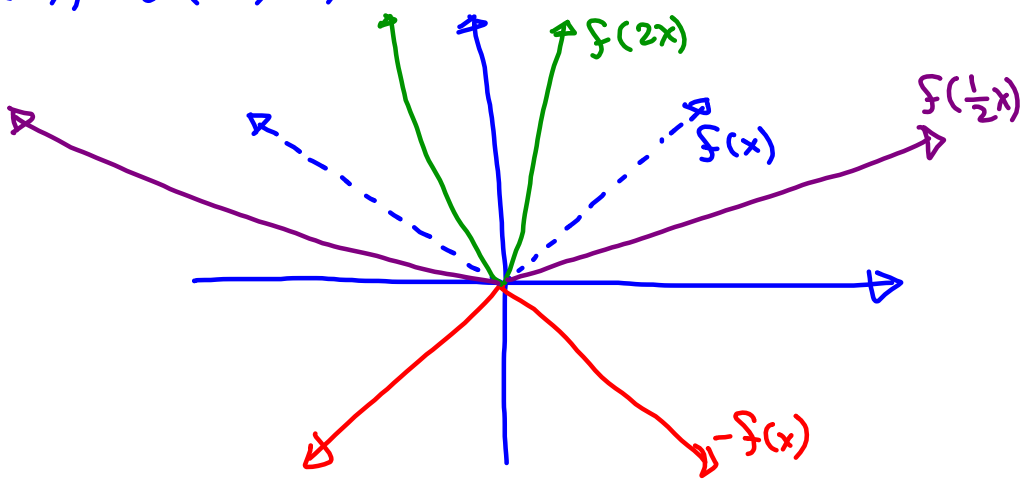


$f(x) = |x|$

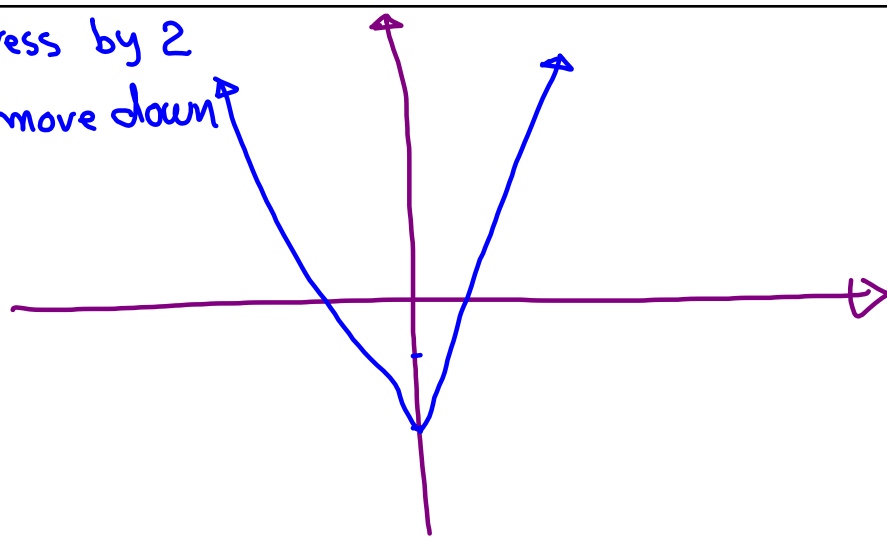


Graph

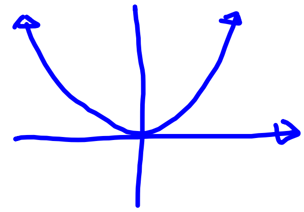
$-f(x)$, $f(2x)$, $f(\frac{1}{2}x)$, $f(2x)-2$



Compress by 2
then move down
by 2



$f(x) = x^2$



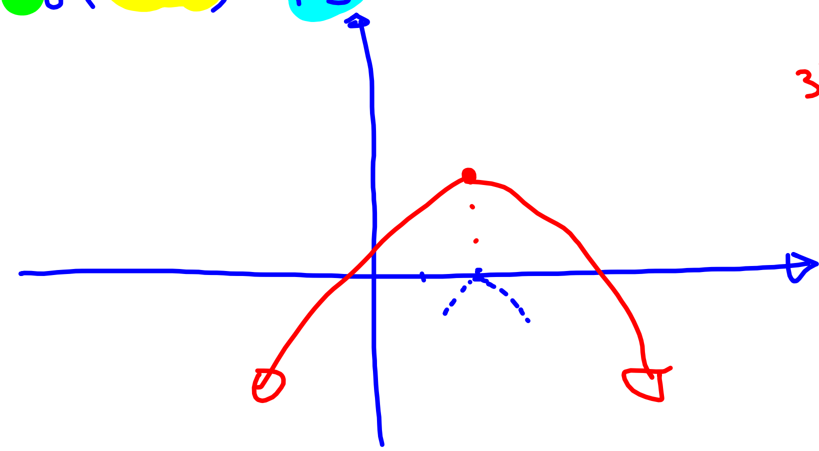
1) take care of x

2) take care of reflection

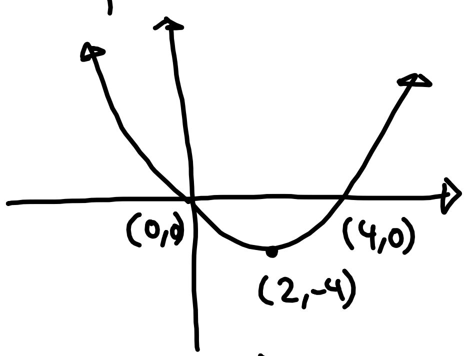
3) take care of y

Graph

$-f(x-2) + 3$

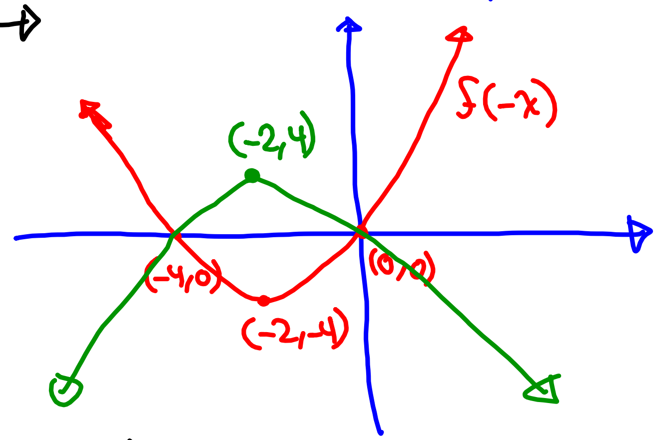


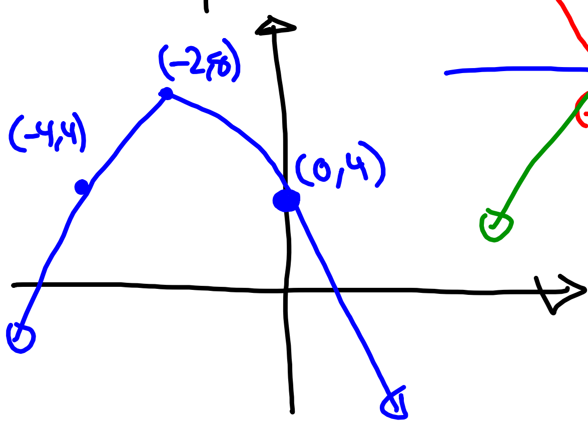
Graph of $f(x) = x^2 - 4x$ is given below



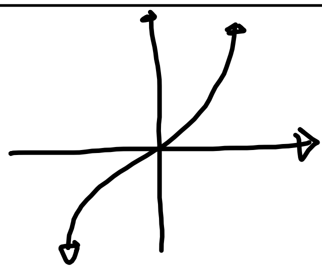
Graph

$f(-x)$, $-f(-x)$, $-f(-x)+4$



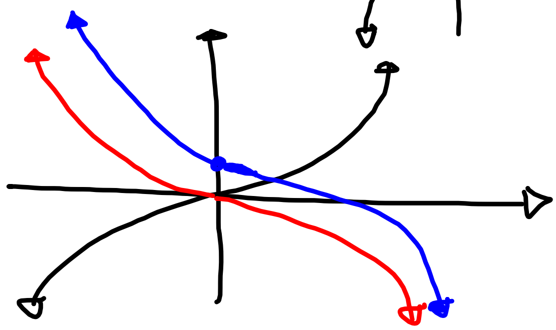


$f(x) = x^3$



Graph

$-f\left(\frac{1}{2}x\right) + 1$



Graph

$f(x) = \sqrt{-x+4} - 1$

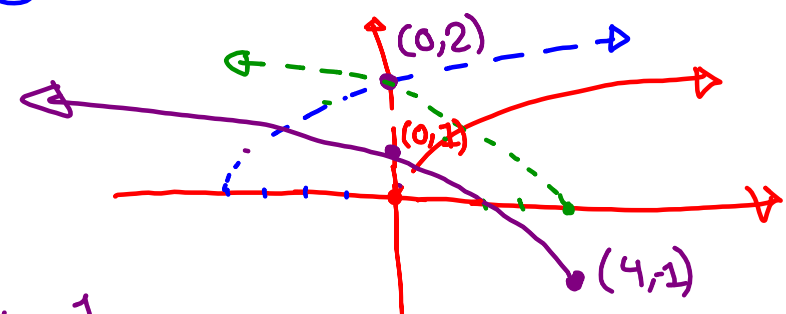
Hint: Start with

$f(x) = \sqrt{x}$

$f(x) = \sqrt{x+4}$

$f(x) = \sqrt{-x+4}$

$f(x) = \sqrt{-x+4} - 1$



Ch. 5 System of linear equations

Subs. method

$$\begin{cases} 3x - \boxed{y} = 5 \\ \boxed{y} = \boxed{2x - 8} \end{cases}$$

$$3x - \boxed{(2x - 8)} = 5$$

$$3x - 2x + 8 = 5$$

$$\boxed{x = -3} \qquad y = 2(-3) - 8$$

$$\boxed{y = -14}$$

System is Consistent.

Equations are independent.

Final ans $(-3, -14)$

Solve

$$\begin{cases} 6x + 2y = 7 \\ y = -3x + 1 \end{cases}$$

$$6x + 2(-3x + 1) = 7$$

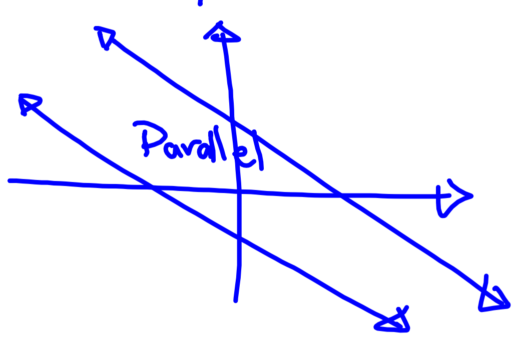
$$6x - 6x + 2 = 7$$

$$2 = 7 \quad \text{false}$$

System $\rightarrow$ inconsistent.

Eqns are independent

No Solution



Elimination method

$$\begin{matrix} 3 \\ 2 \end{matrix}\begin{cases} 3x + 2y = 7 \\ 4x - 3y = 15 \end{cases} \Rightarrow \begin{cases} 9x + 6y = 21 \\ 8x - 6y = 30 \end{cases}$$

$$17x = 51$$

$$\boxed{x=3}$$

$$3(3) + 2y = 7$$

$$9 + 2y = 7 \qquad \boxed{y=-1}$$

$$(3, -1)$$

Solve

$$\begin{cases} 3x - 2y + z = 2 \\ 5x + y - 2z = 1 \\ 4x - 3y + 3z = 7 \end{cases}$$

Final Ans

$$(\ \underset{x}{},\ \underset{y}{},\ \underset{z}{}\ )$$

$$2\begin{cases} 3x - 2y + z = 2 \\ 5x + y - 2z = 1 \end{cases} \Rightarrow \begin{cases} 6x - 4y + 2z = 4 \\ 5x + y - 2z = 1 \end{cases}$$

$$11x - 3y = 5$$

$$-3\begin{cases} 3x - 2y + z = 2 \\ 4x - 3y + 3z = 7 \end{cases} \Rightarrow \begin{cases} -9x + 6y - 3z = -6 \\ 4x - 3y + 3z = 7 \end{cases}$$

$$-5x + 3y = 1$$

Solve $\begin{cases} 11x - 3y = 5 \\ -5x + 3y = 1 \end{cases}$

$6x = 6$

$\boxed{x=1}$

$11(1) - 3y = 5$

$-3y = -6$

$\boxed{y=2}$

$3x - 2y + z = 2$

$3(1) - 2(2) + z = 2$

$\boxed{z=3}$

final Ans

$(1, 2, 3)$

In triangle ABC, The sum of Angles A and B is 20° less than angle C. Angle B is 10° more than twice angle A. find all three angles.

$\begin{cases} A + B = C - 20 \\ B = 2A + 10 \end{cases}$

$A + B + C = 180°$

$\begin{cases} A + B + C = 180 \\ A + B - C = -20 \\ -2A + B = 10 \end{cases}$

$\to 2A + 2B = 160$

$\begin{cases} A + B = 80 \\ -2A + B = 10 \end{cases}$ -1

$$\begin{cases} A + B = 80 \\ 2A - B = -10 \end{cases}$$

$$3A = 70$$

$$\boxed{A = \frac{70}{3}°}$$

$$\frac{70}{3} + B = 80$$

LCD = 3

$$70 + 3B = 240$$

$$3B = 170$$

$$\boxed{B = \frac{170}{3}°}$$

$$A + B + C = 180$$

$$\frac{70}{3} + \frac{170}{3} + C = 180$$

$$\frac{240}{3} + C = 180$$

$$80 + C = 180$$

$$\boxed{C = 100°}$$

Alan has 20 coins. Dimes, Nickels, Quarters only. Total value is $1.95.
# of nickels is the same as total # of quarters and dimes. How many of each?

D → Dimes
N → Nickels
Q → Quarters

$$\begin{cases} D + N + Q = 20 \\ 10D + 5N + 25Q = 195 \quad \div 5 \\ N = D + Q \end{cases}$$

$$\begin{cases} D + N + Q = 20 \\ 2D + N + 5Q = 39 \\ -D + N - Q = 0 \end{cases}$$

$$2N = 20 \qquad \boxed{N = 10}$$

$$\begin{cases} D + N + Q = 20 \\ 2D + N + 5Q = 39 \end{cases}$$

Replace by 10

$$-2\begin{cases} D + Q = 10 \\ 2D + 5Q = 29 \end{cases} \Rightarrow$$

$$\begin{cases} -2D - 2Q = -20 \\ 2D + 5Q = 29 \end{cases}$$

$$3Q = 9$$

$$\boxed{Q=3}$$

$$\boxed{D=7}$$

He has 7 Dimes, 10 Nickels, and 3 Quarters

Looking Ahead

Pascal Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

$A \text{ \& } B \neq 0$

$A \neq -B$

$$(A+B)^0 = 1$$

$$(A+B)^1 = A + B$$

$$(A+B)^2 = A^2 + 2AB + B^2$$

$$(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

$$(A+B)^4 = A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + B^4$$

$$(A+B)^5 = A^5 + 5A^4B + 10A^3B^2 + 10A^2B^3 + 5AB^4 + B^5$$